

ABSTRACT

Travelling Salesman Problem (TSP) involves a salesman visiting a given set of points by starting and terminating at the same point. TSP requires that the salesperson visit each point only once while optimizing the required cost functions, such as time, fuel, fare, etc. In applications such as aerial surveillance, an aircraft acting as the salesman can not change direction instantaneously and have *inherent dynamics restricting the motion*. Also, in applications such as atomic force microscopy raster scanning, the tip of an Atomic Force Microscope (AFM) visits a set of locations on a sample to construct an image of the sample. In raster scanning, the tip of an AFM visits a set of locations by moving from left to right and right to left in a top-down approach. When moving from left to right or right to left, the *dynamics of AFM are approximated with a second-order Linear Time Invariant (LTI) system*. Similarly, when moving from top to bottom, the *dynamics of AFM are approximated with a second-order LTI system*. This thesis considers the TSP assuming the salesman is a dynamical system with inherent dynamics restricting the motion. In particular, we consider Dubins Vehicle, i.e., a system with only forward motion and constant speed, as a salesman in TSP visiting the set of points given in the two-dimensional Euclidean space. The Dubins vehicle as a salesman in TSP is termed the Dubins Travelling Salesman Problem (DTSP). Similarly, we consider another variant of TSP with dynamics in which the second-order LTI system acts as a salesman visiting the set of points in the phase plane, unlike DTSP, in which points are in 2D Euclidean space. We call the TSP for an LTI system the Linear Time Invariant Travelling Salesman Problem (LTITSP).

Owing to the computational complexity involved in TSP, there is *no polynomial time complexity algorithm that provides an optimal solution to the TSP*. For the TSP with system dynamics, the complexity is twofold as compared to the TSP since an additional system's motion constraint is added. TSP has motivated the researchers to opt for approximation algorithms that provide a trade-off between time complexity and solution quality. The addition of system dynamics to TSP increases the complexity of the problem, requiring new and better algorithms that balance time complexity and solution quality.

In this thesis, as a first step towards addressing DTSP, a *worst case upper bound is obtained on DTSP tour length assuming DTSP tour sequence is the same as Euclidean Travelling Salesman Problem (ETSP) tour sequence*. It is noted that, in the worst case, *any algorithm that uses an ETSP tour sequence* is a constant factor approximation algorithm for DTSP. Next, to address DTSP, an algorithm based on dynamic programming called Modified Dynamic Programming Algorithm (MDPA) is presented. MDPA computes the tour sequence and

orientation angles simultaneously. We show that *tour length by MDPA for DTSP is constant times the optimal ETSP tour length*. Next, we present an algorithm called Angle Bisector Algorithm (ABA) to address DTSP. ABA assumes that the DTSP tour sequence is the same as the ETSP tour sequence and computes the orientation angle at each point. ABA works on the principle of angle bisector. We show that the *ABA is a constant factor approximation algorithm when the Euclidean distance between any two points in the given set of points is at least 4ρ where ρ is the minimum turning radius*. To improve ABA, we present a modified version of ABA called Modified Angle Bisector Algorithm (M-ABA). M-ABA also assumes that the DTSP tour sequence is the same as the ETSP tour sequence. However, M-ABA considers more than one orientation angle at each point, resulting in a well-known problem in the literature called Generalized Travelling Salesman Problem (GTSP) or One in a Set TSP. We use *Noon-Bean transformation to convert GTSP to Asymmetric Travelling Salesman Problem (ATSP)* and solve ATSP using the well-established techniques from the literature. The presented approaches are validated through numerical simulations.

In the literature, one of the approaches to addressing DTSP considers a fixed sequence of points in a subset of a given set of points. For a k fixed sequence of points in the given n set of points, the orientation angle at each of these k points is computed so that the path length through this fixed sequence of k points is minimum. In the literature, this problem is termed k -Point Dubins Problem (k PDP). To address DTSP, next, another fixed sequence of k points starting with the second point in the previous sequence along with another point from the unvisited points is considered. This process is repeated till all the points are visited. In this thesis, to address k PDP, an *algorithm called Elimination Algorithm (EA) based on the Groebner basis is presented*. The presented algorithm is validated for the case of $k = 3$ through numerical simulations.

Finally, in this thesis, we consider an LTITSP that is inspired by DTSP. In addressing LTITSP, we first preset a class of points called Class of Optimal Tour (COT) points, for which it is easy to find the time-optimal tour sequence. Using the COT points, we present an algorithm called the Combining Time Optimal Tours (CTOT) algorithm to address LTITSP. The presented algorithm is demonstrated for a particular case of an LTI system, i.e., a double integrator system. For the double integrator case, a *lower bound on tour time is presented, assuming the points are generated according to uniform probability* in the phase plane. Also, we assume that the number of points tends to infinity. The presented approach is validated through numerical simulations.

Keywords: Travelling Salesman Problem, Dubins Travelling Salesman Problem, General-

ized Travelling Salesman Problem, Noon-Bean Transformation, Dubins Vehicle, Angle Bisector, Dynamic Programming, Groebner Basis, k -Point Dubins Problem, Approximation Algorithms.